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CERTIFICATION COURSE

AUTOMATIC CONTROL SYSTEM

LAPLACE TRANSFORM AND TRANSFER FUNCTION

DR ANIL KUMAR

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING



LAPLACE TRANSFORM

- Laplace transform (LT) is a widely used integral transform with many applications in science and engineering.
- It maps differential or integro-differential equations in the 'time' domain into polynomial equations in 'complex frequency' domain.

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\mathcal{L}^{-1}F(s) \equiv f(t) := \frac{1}{2\pi \cdot j} \int_{c-j\infty}^{c+j\infty} F(s) \cdot e^{st} ds$$

LAPLACE TRANSFORM

- Laplace transform (LT) of some common functions and operations

If $L\{f(t)\} = F(s)$, then

$$L \frac{d^n}{dt^n} f(t) = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

Foreg. Let $n = 1$

$$L \left\{ \frac{d}{dt} f(t) \right\} = sL\{f(t)\} - f(0)$$

If $L\{f(t)\} = F(s)$, then

$$L \left[\int \int \int \int \dots \int f(t) dt^n \right] = \frac{1}{s^n} L\{f(t)\} + \frac{f^{(n-1)}(0)}{s^n} + \frac{f^{(n-2)}(0)}{s^n} + \dots + \frac{f^{(0)}(0)}{s}$$

Foreg. Let $n = 1$

$$L \left\{ \int_0^t f(t) \right\} = \frac{1}{s} L\{f(t)\} + \frac{f'(0)}{s}$$

$f(t)$	$\mathcal{L}[f(t)]$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$\frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{s^{n+1}}$
$e^{at} \cdot \frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{(s-a)^{n+1}}$

TRANSFER FUNCTION

- Transfer functions are commonly used to characterize the input-output relationships of components or systems that can be described by linear, time-invariant differential equations
- Transfer Function (TF) of **a linear, time-invariant, differential equation system** is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, under the assumption that *all initial conditions are zero*



TRANSFER FUNCTION

- Transfer Function (TF) is a property of the system itself. It is independent of the input. However, it does not provide any information concerning the physical structure of the system.
- TF is defined only for a linear, time-invariant system. It is not defined for nonlinear systems



REFERENCE BOOKS

- Norman S. Nise, Control Systems Engineering, Wiley, 2013
- Katsuhiko Ogata, Modern Control Engineering, Prentice Hall, 2010.

